A critique of using real options pricing models in valuing real estate projects and contracts

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Abstract
Discounted cash flow (DCF) models have been criticised for using risky discount rates and subjective estimates of future cash flows. In addition, DCF models do not incorporate valuations of implicit options imbedded in capital projects. Recently, researchers have applied real options pricing models to evaluate real estate projects and contracts. However, the required assumptions and criteria for using these models may be absent in real estate projects, which raises the question whether these models produce better results or create more uncertainties for end users. This paper contains a review of the conditions and methods that have been proposed for applying real option models in real estate valuations.

INTRODUCTION
Sirmans (1997) suggested that the traditional discounted cash flow (DCF) model may be insufficient for evaluating real estate projects. His observations supported previous critiques of DCF analysis (Hayes and Abernathy 1980; Hodder and Riggs 1985) that questioned the selection process of discount rates and the inability of DCF analysis to include options in the valuation of a project. These researchers argued that failing to include changes in future cash flows caused by options that exist in most capital projects bias the results of a DCF analysis. The introduction in the 1970s of stochastic calculus in the economic and finance literature created a renaissance in valuing financial option contracts. Many researchers applied these financial option-pricing models to capital budget projects. As a result of the gain in popularity of this new area of ‘real option’ analysis among finance researchers during the early 1990s, several research papers in real estate valuation (Quigg 1995; Buetow and Albert 1998; Hendershott and Ward 2000; Holland et al. 2000) have included real option valuation models.

Yet the question remains whether these complex option pricing models have reduced uncertainties or whether their underlying assumptions add uncertainty about the results. Prior to the real options renaissance, DCF models provided the primary tool for evaluating the feasibility of corporate projects. Within a DCF...
model, an analyst must develop anticipated cash flows for a project. In addition, the analyst selects a discount rate that represents the riskiness of the project and the analyst’s cost of capital. The analyst’s experience, perceptions, biases and subjectivity play a role in developing both the future cash flows and the discount rate. Including option-pricing models adds another level of assumptions and complexity to the valuation task.

Survey results of corporate capital budgeting practices found that most companies currently rely on less complex models for evaluating projects. A recent survey of companies in the S&P industrial index (Farragher et al. 1999) showed that most of the respondents relied on traditional DCF models and that sensitivity analysis and intuition play a key role in adjustments for risk. Also, Block’s (1997) survey showed that smaller companies (annual sales less than $5,000,000 and fewer than 1,000 employees) typically did not employ DCF models when evaluating projects. Given these data, does the real estate industry need a complex valuation model that may be difficult for the user to understand and may not reduce uncertainty of the results? The recent failures of stochastic option pricing models in security and commodity markets, for example Long Term Capital Management, makes one suspicious that these models should be adapted to real estate markets that lack some of the efficiencies of publicly traded securities.

The content of this paper includes a review of proposed techniques for valuing real options in real estate problems and to identify options that occur with real estate assets. The author provides a critique of the applicability of these techniques to real world problems and notes limitations associated with assumptions required for solving the complex mathematics incorporated in the models. Also, a hypothetical example is provided. With the growing body of research in this area, it is critical to begin examining the applicability and practicality of these models to the real estate industry.

OPTIONS IN REAL ESTATE PROJECTS AND CONTRACTS
Several researchers (Kemna 1993; Trigeorgis 1993) have identified options embedded in capital projects that apply to real estate valuations. These options result from explicit terms and conditions in a contract or because of a strategic advantage held by the principal. Both Kemna and Trigeorgis used the timing of extracting oil reserves as an example of an option to defer a decision until additional information evolves that makes extraction profitable. This equates to a real estate developer’s option to delay construction until demand improves in the local economy. Other options identified by these authors included the option to expand or contract the production capacity of a plant after start up, the option to shut down or abandon a project after start up in response to unfavourable economic conditions, and the option to switch production inputs and outputs in response to changing consumer preferences.
demands. All of these options exist in some form in real estate projects or contracts. In addition, multiple options (both puts and calls) that interact with one another may exist within a single project. Each of these authors presented theoretical models for assessing the value of these options in capital projects. Current research on real estate options has focused on mortgages, development rights and lease contracts.

Most real estate assets include implied or explicit options. For example, mortgage contracts in the USA have explicit call options that allow a debtor to prepay the balance of the mortgage. The development of public securitised secondary markets for mortgages and uniform mortgage contracts enabled researchers to apply financial options pricing models to value the embedded options in mortgages. Mortgage investors have a substantial need for valuing prepayment options because the option has a significant negative effect on the market value of mortgage backed securities (MBS) when market interest rates decline. In addition, other socio-economic factors, such as migration rates, play a role in prepayment rates. As a result, investment bankers and researchers (Green and Shovan 1986) have developed forecasting models to estimate prepayment rates. Other researchers (Brennan and Schwartz 1985; Kau et al. 1992) have applied option methodology to valuing mortgages containing prepayment risks.

Owners of vacant land have implied options associated with the timing of development (ie delay development) or options to shut down or abandon a development after start up. Quigg (1995) presented a method of valuing these options as perpetual American options. She based her model on the same theoretical calculus and partial differential equations (PDEs) used in the Black-Scholes option-pricing model. Some of Quigg’s model assumptions include complete real estate markets without arbitrage opportunities, a publicly traded portfolio with perfect or constant correlation with the building value, and that the average return of the portfolio is known and constant with a constant volatility (variance) over time. Her model’s results suggest that the value of the option to wait exceeds the value of expected rents for a building at the decision date. Consequently, the owner of vacant land should defer development and sell the vacant land (and its option) to another investor to maximise the benefit of the land.

A typical real estate lease contract contains many options to both the lessor and lessee. Posner (1993) identified several options inherent in leases. Examples of lessee call options are expansion options and renewal options. Mark to market clauses and subleasing restrictions represent call options owned by the lessor. Limited liability leases and cancellation clauses represent put options held by the lessee. Posner suggested that these options should be included in valuing leases from both the perspective of the tenant and landlord.
Sirmans and Miller (1997) also agreed that options in lease contracts represent value that must be addressed when negotiating and accounting for real estate leases. Hendershott and Ward (2000) examined pricing leases that contain both an option to renew on terms indexed to inflation and an overage option indexed to a tenant’s sales. The tenant receives the renew option, which is a call option, and will exercise the option if rental rates exceed the inflation index. Simultaneously, the landlord/lessor receives a call option from the tenant that permits the landlord to receive a percentage of the tenant’s gross sales above a specified threshold. The authors’ model used a continuous time framework and assumed that rents follow a Geometric Brownian Motion (GBM) with constant mean and variance over time. Similarly to other stochastic continuous time models, the authors assumed a parallel portfolio of traded assets that is perfectly correlated with a subject property’s rental rates. Results of their work showed that the existence of these two options effectively cancel each other out. Hence, the authors suggested that pricing of these simultaneous options may be less important in valuing the lease.

Buetow and Albert (1998) also developed a model for evaluating leases containing call options that give the lessee the right to renew a lease at a rate indexed to inflation. Similarly to previous continuous time asset pricing models, the authors assumed that rents follow a GBM with known mean and variance. Since the strike price (rent indexed to inflation) of the option is stochastic instead of fixed, a closed formed solution for the resulting PDEs cannot be obtained. Hence the authors used a numerical analysis to approximate solutions to the equations. The authors demonstrated their model by valuing renew options in several urban areas using quarterly rent data from the National Real Estate Index. The authors concluded that the complexity of implementing the model make it inappropriate for valuing leases for individual properties, but that the model may be valuable for determining option values within a market.

**VALUING REAL OPTIONS**

Teisberg (1995) and Lander and Pinches (1998) provided a comprehensive discussion of methods for evaluating capital projects containing options. Generally, techniques fall into either discrete time models or continuous time models. Each technique adds another layer of assumptions and complexity to a conventional DCF model. Most of these techniques require a significant increase in the user’s mathematical skills and economic knowledge. The following gives a brief discussion of these techniques and the required assumptions for the models to be valid.

Binomial, lattice and dynamic discounted cash flow (DDCF) models proceed in discrete time. All three of these models can be viewed as decision tree processes. At each point in time in which
an option or decision choice exists, the user splits the tree into two or more paths. Copeland et al. (2000) provided a detailed example of using decision trees for valuing a capital project. In a binomial model, the user assigns a probability (p) to an event increasing and the amount of the increase (u). Similarly, the user assigns a value of a decrease (d), that has a probability of occurring equal to 1–p. A risk-free rate is used as the discount rate. A solution is obtained by folding the tree back using recursion. The model assumes that the risk-free rate is constant, that the underlying asset for the option exists, is measurable, and has a constant variance, and that the u and d parameters for the asset are constant. Estimating values for p, u and d may be relaxed in cases where the user assumes that the underlying asset has log normal distributed returns and follows a well defined mathematical function, such as a Geometric Brownian Motion. In this case, only the variance of the asset’s returns is required. Hodder and Riggs (1985) suggested modifying the traditional DCF analysis to include future management decisions or choices in response to changes in events. They argued that capital projects should be viewed in phases, with each phase having different risks and cash flows. Their work laid the basis for creating a DDCF analysis. Teisberg (1995) provided further details of implementing a DDCF analysis which, similarly to a binomial model, require the user to develop a decision tree with branches at each point in time in which the user has an opportunity to make changes in the project. The user develops estimates of the cash flows that occur along each branch and the probability that the event will occur. The present value of the future cash flows is computed using a discount rate equal to the opportunity cost of capital. Since risks may be unequal between project phases, the user can risk adjust the discount rates for different time periods. A DDCF model requires the user to carefully consider the future cash flows of the project because of the different choices that may occur in the future. Selection of the probabilities of each branch of the decision tree is subjective and should be closely scrutinised by the analyst. In addition, the DDCF model, similar to a traditional DCF analysis, requires the user to define discount rates that reflect the project’s cash flow risk and the firm’s cost of capital.

Continuous time models for valuing options typically follow the Black-Scholes model (1973) or use a system of PDEs with boundary conditions. Both methods assume that the option has a traded twin asset that follows a known well-defined process, such as a GBM. In addition, both methods usually assume a log-normal distribution of the twin asset’s returns and that the mean return and variance of the returns are known and constant. Other assumptions of these models include complete markets, no arbitrage opportunities and a constant risk-free rate. If complete markets and no arbitrage opportunities cannot be assumed, then an alternative is to assume a risk-neutral firm or that risk can be fully diversified. When the option has state...
dependent or time dependent input parameters (eg the strike price follows a stochastic process), or multiple simultaneous options exist, a system of PDEs must be developed that models the changes in the option value relative to changes in the model’s variables. Furthermore, the user must develop the boundary conditions for the equations. Typically, due to the complexity of the PDEs, the user cannot obtain a closed form solution, so the user must select a numerical technique, such as a finite difference method, to approximate a solution to the equations.

DDCF models obtain a value for the capital project that includes the options. Estimated cash flows at each phase of the project contain an implied future value of the option. Hence, the resulting value represents the present value of the project’s cash flows that includes the implied options. Some applications of binomial, lattice, Black-Scholes and PDE models only compute values for a project’s options. The option values can be added to a traditional DCF analysis to obtain an overall value of the project. When multiple options exist, they are either valued independently or combined. However, simultaneously valuing interdependent options in the same PDE model may be difficult or impossible to resolve. When one of the above models only values the options, the user must still prepare forecast cash flows and estimate a discount rate to value a capital project.

CASE STUDY
Suppose a real estate developer can presently erect an office building for $225 a square foot. The developer estimates rental rates one period from the present for the completed office building at $22 per square foot in perpetuity. In addition, the developer estimates a risk adjusted discount rate (or capitalisation rate) for the building project of 11 per cent. Following traditional DCF analysis the developer would compute the NPV of the project as −$25.00 using the following equation:

\[
\text{NPV}_{\text{wo/Option}} = -225 + \sum_{t=1}^{\infty} \frac{22}{(1.11)^t} = -25.00
\]  

Although the developer knows with certainty that the building will cost $225 per square foot, the future rental cash inflows are less certain. Suppose the developer believes that rental rates have an equal probability of equalling $14 or $30 per square foot in one year. If the developer can delay construction for one time period, he or she has the opportunity to gain more information on the future market. Theoretically, this delay option has value to the project. Valuing the delay option using a simple dynamic discounted cash flow model requires computing the present value of the weighted average of the two possible future cash inflows.
NPV_{w/Option} = .5 \left[ \text{MAX} \left( -\frac{225}{1.11} + \sum_{t=2}^{\infty} \frac{14}{1.11^{t}}, 0 \right) \right] \\
+ .5 \left[ \text{MAX} \left( -\frac{225}{1.11} + \sum_{t=2}^{\infty} \frac{30}{1.11^{t}}, 0 \right) \right] \\
= .5[0] + .5[43.00] \\
= \$21.50

The MAX function returns a value of zero when the future rental cash flows are less than the cost of construction. This is the case when rents are only $14, the developer would elect not to develop, essentially letting the option expire. The implied value of the delay option is the difference between the project with the option and without the option, $46.50 = $21.50 – (–$25.00).

This analysis implies that the developer should delay construction, if he or she has the option of delaying. Alternatively, the developer could consider purchasing a development option from a land owner and exercise the option only if rents increase. Or a real estate investor could purchase an option to buy the completed building from a developer at the end of one period. The investor would exercise the option if the rental rates increase. Equation 2 provides a simplified method for estimating the values of these options. Of course, delaying entry into a market may have unknown and unwanted consequences that are difficult to model. In addition, as this simple model shows, several other conjectures were required to evaluate the delay opportunity.

Now suppose that today an investor can enter into a contract to purchase in one period a building for $250 per square foot. By signing the contract today, the investor has committed to a cash outflow of $250 one period from today. Alternatively, the investor prefers purchasing a call option contract that allows them to purchase the building at the end of the period for $250. What should the investor expect to pay for this right to delay the decision to purchase the building for one period? Evaluating the option using a discrete time binomial model requires a twin asset, a risk-free asset and the assumption of an arbitrage free market. The option can then be valued under the assumption that a portfolio consisting of the twin asset and the risk-free instrument can be constructed that has identical payoffs to the option.

Copeland et al. (2000) suggest using the building that the investor is considering purchasing (or a close substitute) as the twin asset. Suppose that today investors would pay $225 per square foot for the building on anticipation that in one period there is an equal probability that the building will be worth either $165 per square foot or $350 per square foot. Investors’ anticipated return of 14.44 per cent from this investment is found by solving for k in the following formula.
Note that the present value of the $250 exercise price discounted at the risk-free rate equals $229.36, which exceeds the current value of the building’s expected future cash flows of $225. Hence, the value of entering into the contract that commits the investor to purchasing the building in one period for $250 equals $-4.36. The negative value implies that the investor would not commit today to purchase the building one period from today for $250 per square foot. However, the investor may be interested in purchasing the option to delay this decision for one period.

The payoff on the option at expiration equals either $100 or $0 depending on whether the building’s value increases to $350 per square foot or decreases to $165 per square foot. Under the assumption of no arbitrage opportunities and a risk-free rate of 9 per cent, a portfolio consisting of an investment in the twin security (the building of interest in this case) and the risk-free instrument that replicates the option would have the following payoffs.

\[ N \times 350 + B(1 + r_f) = 100 \]
\[ N \times 165 + B(1 + r_f) = 0 \]  

where \( N \) represents the amount of the twin asset to purchase and \( B \) represents the amount of the risk-free security to purchase or sell. Solving the above equations yields a value of .541 for \( N \) and $-81.82 for \( B \). The value of the option equals the value today of the replicating portfolio less the present value of committing today to purchasing the building in one period for $250.

\[ \$44.15 = .541(\$225) - \$81.82 - (\$-4.36) \]  

The above examples demonstrate the use of simple discrete time option models to price a delay option in a real estate investment decision. The addition of a single option in the project required the analyst to include several assumptions to produce a value. These assumptions may be difficult to obtain with a reasonable certainty.

**CRITIQUE OF METHODS FOR EVALUATING REAL OPTIONS IN REAL ESTATE**

Static DCF analysis has been criticised (Hodder and Riggs 1985, Sirmans 1997) for providing an incomplete or misleading investment decision. Criticism has focused on the DCF’s discount factor, subjective projection of future cash flows, and the omission of options in valuing the project. Researchers have questioned the validity of using a constant discount rate for the length of the project. They have argued that the discount rate should be adjusted...
relative to time for effects from inflation and specific risks associated with individual cash flows. Obviously, selection of the appropriate discount rate is critical in a DCF analysis because of the sensitivity of the present values of the future cash flows to changes in the discount rate. Furthermore, the DCF model depends on a manager to forecast the amount and timing of future cash flows accurately. In the presence of competing real estate projects, a manager may provide cash flow projections lacking in objectivity that improve the present worth of his or her project. Scrutiny by an independent reviewer or board can facilitate minimising bias in forecasts.

Omission of option valuations represents a severe shortcoming of static DCF analysis. The traditional DCF model lacks provisions for including future changes in a project’s cash flows that result from choices made by managers in response to changing economic conditions. Thus, static DCF analysis requires another mechanism for including options in the valuation process.

Mortgages and mortgage-backed securities valuations have received significant attention because of their embedded prepayment and default clauses. With the growth of mortgage secondary markets, institutional investors have developed many models for predicting prepayment and default rates in mortgage pools. Standards enforced by the Federal National Mortgage Association (FNMA) and Federal Home Loan Mortgage Corporation (FHLMC) have transformed mortgages into near-homogeneous assets, which minimise changes in assumptions to apply a valuation model over different mortgage pools. In addition, mortgage-backed securities trade in robust public markets that provide a wealth of pricing information and products to investors. Hence, there exists abundant information for investors to use in calculating measures of average return and volatility. Option valuation models (discrete or continuous time) require these measures and assumptions to produce valid prices. Hence, it appears that application of these techniques to mortgage-backed securities may be appropriate. However, the same cannot be claimed for other real estate assets, such as leases or development options, because of the underlying assumptions of these option valuation models.

Recent articles (Quigg 1995; Sirmans 1997; Buetow and Albert 1998) have suggested using security option pricing models (binomial, Black-Scholes, or Black-Scholes variations) to value real options embedded in real estate assets such as leases and development options. But, before applying these models researchers should ensure that the assumptions of the models exist in the application. This may require empirical observations to validate the existence of a model’s requirements. The security option pricing models discussed in this paper usually require the following assumptions:

- Complete markets.
- No arbitrage opportunities or fully diversifiable risk.
• An observable underlying (twin) asset that has perfect correlation or constant correlation with the asset in question.
• Log-normal returns for the underlying asset.
• A known and constant mean return and variance of the underlying asset that is identical to the asset in question.
• A known and constant risk-free rate.
• A well-defined mathematical process that the underlying asset follows in time, such as a GBM process.

These assumptions should be validated before proceeding with one of the models. If an analyst cannot validate the model’s assumption then the analyst should proceed to investigate the sensitivity of the results relative to the model’s underlying assumptions.

Most practitioners agree that real estate markets have significant inefficiencies. Private transactions, infrequent trades, costly fees, information asymmetries, and many local markets create these inefficiencies and add to real estate’s illiquidity when compared to stocks or bonds. Hence, real estate markets may not provide the risk-free arbitrage environment required for option-based modelling (Pagliari and Sanders 1997). Real estate markets do not provide the investor with the ability to fully diversify risk. This may make it inappropriate to assume risk neutrality and use a risk-free rate in evaluating the option.

This is true for options in real estate leases and purchases because they result from private transactions. Thus, a reliable source for market data on real estate transactions does not exist. This presents significant problems when selecting an underlying asset. Option-based models place a burden on the analyst to select an underlying asset or portfolio that is perfectly correlated with the real estate asset. In addition, the analyst must assume log-normal returns and that the average return for the underlying asset and its volatility as measured by its variance reflect the behaviour of the real estate asset. Usually these data are not available, so the analyst must assume that the underlying asset has characteristics that fit the model’s assumptions. Past research (Laughton and Jacoby 1995) showed that the option valuation was biased from changes in the assumed underlying mathematical process.

Problems also exist when applying binomial models. In this case, the analyst must know u, d, p and a risk-free rate that allow convergence of the binomial tree. Without a reliable and observable underlying asset an analyst must estimate these parameters. Misstating the parameters significantly affects the results of the binomial model. Hence, the analyst may need to be prepared to perform sensitivity analysis to determine the impact of the parameter selection on the valuation. Furthermore, what evidence does the analyst have that these parameters remain constant? In the real world, investment returns, volatility and payouts are often state and/or time dependent.
Most real estate transactions also contain multiple, interdependent options. For example, leases may contain renewal options, expansion options and sublet options. The values of these options may be dependent on each other. However, the required mathematics to simultaneously model multiple options quickly becomes intractable. As a result, simplifying assumptions (Hendel and Lizzeri 2001) must be made to reduce the number of options. In addition, the exact influence that one option has on another option may be impossible to measure or predict (Trigeorgis 1993).

Past researchers in real estate option valuations (Pagliari and Sanders 1997; Buetow and Albert 1998) have suggested using indexes based on REIT stock performance or published survey data on leasing rates to act as a proxy for the underlying asset. However, problems exist when using these data because of the nature of real estate assets and markets. REIT returns may not be consistently correlated with returns from direct ownership in real estate. Several articles (Gyourko and Kiem 1992; Han and Liang 1995, Oppenheimer and Grissom 1998) have shown that REIT returns may have stronger correlations with stock market returns than with indexes that serve as proxies for real estate returns. Survey data may offer some relief, but still do not reflect actual market returns. However, a bigger problem may exist from assuming that an index based on aggregate data applies to an individual real estate project. Asset uniqueness represents a characteristic that distinguishes real estate/property from other asset classes such as stocks and bonds. It may be impossible to find an observable underlying asset that mimics a specific real estate project. Consequently, when using an option-based model it takes a leap in faith by an analyst (and the analyst’s principals) that the underlying security fits the definition of the model and relates to the specific real estate project. This may explain why much of the literature (eg Kemna 1993) on real option evaluation has focused on resource extraction where an adequate proxy exists for an underlying security.

A final consideration that must be addressed when applying a security option-based model to real estate is the loss of earnings and the time to fulfil exercising the option (eg constructing a building). Loss of earnings is similar to options on securities that pay dividends. When a developer elects to delay construction, the developer gives up the rents that would have been realised from a completed building. This is similar to foregoing dividends that would have been received if the investor owned the stock instead of the option. Unlike financial options, when a developer decides to exercise his or her option and begins construction, a significant time lag exists between completion of the project and resulting cash flows. Quigg (1995) accounted for some of these characteristics, but the resulting model required further assumptions to make it somewhat tractable. Lander and Pinches (1998), Teisberg (1995) and Trigeorgis (1993) warned that accounting for these issues is often difficult and quickly leads to intractable mathematical models.
CONCLUSIONS
Without question, static DCF analysis omits pricing explicit or implied options embedded in real estate assets. However, substituting security option pricing models for static DCF may introduce more unknowns than originally existed. This results from the absence of a reliable twin security for real estate assets that can be used in the option valuation model. Furthermore, the existence of interrelated options in a single real estate project makes an option pricing model’s mathematics intractable. Finally, unlike financial assets, real estate assets lack uniformity, hence a unique mathematical model may be required for every real estate asset valuation. This adds significantly to the cost of using security option pricing models for valuing real estate.

When an analyst elects to proceed with a security option model, he or she should examine the results for sensitivity to changes in the model’s assumptions and parameters. Teisberg (1995), Lander and Pinches (1998) and Copeland et al. (2000) suggested alternatives to using security option models for valuing real options in capital projects. Their alternatives (DDCF analysis and influence diagrams) appear applicable to real estate assets because they do not depend on an observable underlying asset. Furthermore, the solutions to these procedures do not require the intense mathematical skills required to solve a security option pricing model. Also, both alternatives are intuitive and easier to use for diagramming future cash flows and benefits of a project.

References


